Name: ..……………………………………………………………………………….

Student Number: ………………………………………………………………………..

Test 6 on WPPH16001.2019-2020
“Electricity and Magnetism”

Content: 12 pages (including this cover page); **5 questions**

Wednesday 17 2020; **online**, 8:30-11:30

- Write your full name and student number on **each** page you use
- Read the questions carefully. Read them one more time after having answered them.
- Compose your answers is such a way that it is well indicated which (sub)question they address
- Upload the answer to each question as a **separate pdf file**
- Do not use a red pen (it’s used for grading)
- Griffiths’ textbook, lecture notes and **your** tutorial notes are allowed. The internet, mobile phones, consulting and other teamwork are not allowed (and considered as cheating)

Exam drafted by (name first examiner) Maxim S. Pchenitchnikov
Exam reviewed by (name second examiner) Steven Hoekstra

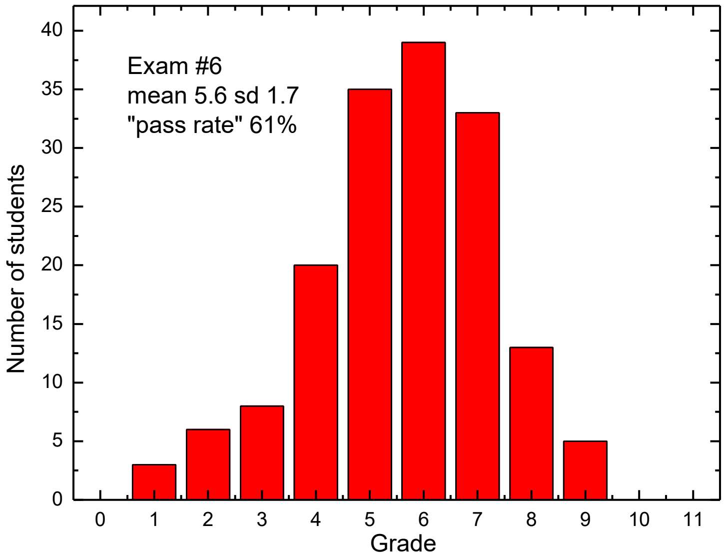
For administrative purposes; do NOT fill in the table

The weighting of the questions:

	Maximum points	Aver. Points scored
Question 1	10	4.9
Question 2	15	6.5
Question 3	10	7.5
Question 4	10	4.9
Question 5	15	9.2
Total	**60**	**31**

Grade = 1 + 9 x (score/max score).

Aver. Grade: 5.6

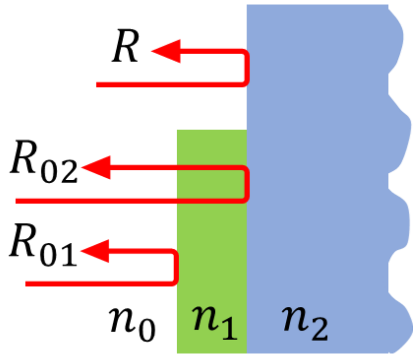


General remarks:

- Forgetting the vector sign (which is a mistake per se) leads to even more mistakes when calculating cross-products.
- You should express your answers in the original variables given!

Question 1. (10 points)

The upper part of the figure shows reflection of an electromagnetic wave at an interface between two materials with refractive indices n_0 and n_2 under normal incidence ($\mu_1 = \mu_2 = \mu_0$). A thin layer of material with refractive index n_1 (shown in green) is applied upon the surface of the n_2 material (the lower part of the figure). The wave now reflects twice: once from the surface between the n_0-material and the thin layer (reflection coefficient R_{01}), and once from the layer-to-n_2-material interface (reflection coefficient R_{02}).



1. Show that the optimal refractive index n_1 of the layer (for given n_0 and n_2) that minimizes the double-reflection $R_{01} + R_{02}$, is $n_1 = \sqrt{n_0 n_2}$.

Tip: upon derivation, consider reflection coefficients for *amplitudes* of electric fields rather than for *intensities* of the waves (i.e. $\sqrt{R}$ instead of R). Also consider the transmission coefficient at the $n_0 \leftrightarrow n_1$ interface as $T_{01} \cong 1$, i.e. do not include multiple reflections. (5 points)

2. What is the optimal refraction index n_1 for the visible light and the air ($n_0 = 1$) to glass ($n_2 = 1.5$) interface? (1 point)

3. Calculate the value of the total reflection coefficient $R = R_{01} + R_{02}$ for the system air+layer+glass (now for intensities). Did you manage to reduce reflection as compared to the air+glass system, and if so, by which factor? (2 points)

4. Now calculate the transmission of both air+layer+glass and air+glass systems. Which one is higher? (2 points)

Tip: Such a layer is called an "antireflection coating" not for nothing!

Answers to question 1 (10 points)

1. $\tilde{E}_{0_R}^{\ (0-1)} = \dfrac{n_0 - n_1}{n_0 + n_1} \tilde{E}_{0_I}$; $\tilde{E}_{0_R}^{\ (1-2)} = \dfrac{n_1 - n_2}{n_1 + n_2} \tilde{E}_{0_I}$

Sum of the two reflections: $\dfrac{\tilde{E}_{0_R}^{\ (0-1)}}{\tilde{E}_{0_I}} + \dfrac{\tilde{E}_{0_R}^{\ (1-2)}}{\tilde{E}_{0_I}} = \dfrac{n_0 - n_1}{n_0 + n_1} + \dfrac{n_1 - n_2}{n_1 + n_2}$ (1 point)

Calculating the derivative with respect to n_1 and equalizing it to zero:

$$\frac{-(n_0 + n_1) - (n_0 - n_1)}{(n_0 + n_1)^2} + \frac{(n_1 + n_2) - (n_1 - n_2)}{(n_1 + n_2)^2} = 2\frac{-n_0}{(n_0 + n_1)^2} + 2\frac{n_2}{(n_1 + n_2)^2} = 0$$

(2 points)

$-n_0 n_1^2 - 2n_0 n_1 n_2 - n_0 n_2^2 + n_2 n_0^2 + 2n_2 n_0 n_1 + n_2 n_1^2 = 0$ (1 point)

$n_1^2(-n_0 + n_2) = n_0 n_2 (n_2 - n_0)$; $n_1 = \sqrt{n_0 n_2}$ (1 point)

2. $n_1 = \sqrt{1.5} = 1.22$ (1 point)

3. $R_{01} = \left(\dfrac{1 - 1.22}{1 + 1.22}\right)^2 = 0.0099$; $R_{02} = \left(\dfrac{1.22 - 1.5}{1.22 + 1.5}\right)^2 = 0.011$; $R = R_{01} + R_{02} \cong 0.02$

(1 point)

As the direct reflection at the air-glass interface amounts to 0.04, there is a factor of 2 reduction in reflectivity. (1 point)

4. $T = 1 - R$ because the total energy must be concerved (1 point)

Transmission air+layer+glass 0.98; transmission air+glass 0.96 so that the transmission air+layer+glass is enhanced! (1 point)

Typical mistakes:

Q1.1. Taking the two reflectances to be equal for some unknown reason

Q1.4. Adding T coefficients so that T>1, which is clearly impossible by definition

Q1. Luck of understanding where you can use the approximation $\mathrm{T}_{01} \cong 1$, (i.e. do not include multiple reflections) and where you cannot ($\mathrm{T} = 1 - \mathrm{R}$ because otherwise your answer doesn't make sense)

Notes:

1. Lord Rayleigh discovered this phenomenon in 1886 as he noticed that a thin layer (such as water) on the surface of glass can reduce the reflectivity of the visible light.

2. The actual working principle also includes the effect of interference which you could have accounted for if you explicitly considered propagation of the wave forward to and back from the $n_1 - n_2$ interface and then calculated the total reflection as $\propto |E_{01} + E_{02}|^2$ (spoiler: the wave reflected from the second interface gains a "phase" factor of $\exp(2ik_1 d)$, where d is the layer thickness). In this case, you can achieve zero reflection for a certain wavelength – but let us leave this for the *Waves and Optics* course.

3. Unfortunately, no technologically relevant material has the desired refractive index so that magnesium fluoride (MgF_2) is often used for such a layer even though its index is $n_2 = 1.38$. You can calculate yourself if you manage to reduce the refraction loses and by what factor.

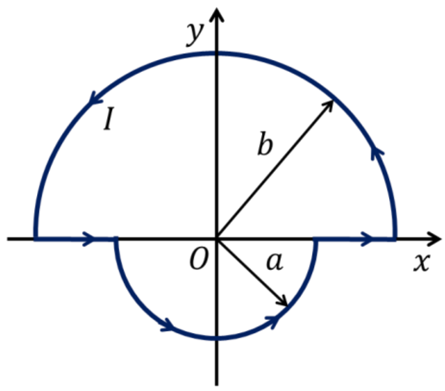
4. Those of you who wear glasses, can see such a layer if you look at the lenses at grazing incidence: they are slightly coloured (typically, bluish). This results from interference caused by the antireflection layer(s).

5. Magnesium fluoride is also hard-wearing so that it automatically provides an anti-scratch coating for plastic lenses (as e.g. in ordinary glasses).

Question 2. (15 points)

A piece of wire bent into a loop, as shown in the figure, carries a current that increases linearly with time:

$I = kt \; (-\infty < t < \infty)$



1. Show that the retarded vector potential $\vec{\mathbf{A}}$ at the center O is

$\vec{\mathbf{A}} = \frac{\mu_0 kt}{2\pi} ln(b/a)\,\hat{\mathbf{x}}$ (7 points)

Tip 1: don't forget about retardation!

Tip 2: you might find useful the expression of $d\vec{\mathbf{l}}$ in cylindrical coordinates:
$d\vec{\mathbf{l}} = s\, d\hat{\boldsymbol{\varphi}} = s(-\sin\varphi\,\hat{\mathbf{x}} + \cos\varphi\,\hat{\mathbf{y}})$ (if $s = const$)

2. Find the electric field $\vec{\mathbf{E}}$ at the center. (1 point)

3. We can't compute $\vec{\mathbf{B}} = \boldsymbol{\nabla}\times\vec{\mathbf{A}}$ to get $\vec{\mathbf{B}}$ at the center O because we know $\vec{\mathbf{A}}$ at one point only (the center). Compute $\vec{\mathbf{B}}$ at the center O using Jefimenko's equation for the case $b = a$.
(5 points)

4. Compare your result with the one calculated earlier in the course on basis of the Bio-Savart law (Eq.5.41 with $z = 0$). Why are the two results identical despite the fact that the Bio-Savart law is valid for *constant* currents while here the current does change in time? (2 points)

Answers to question 2 (Problem 10.12 modified, 15 points)

1. $\vec{\mathbf{A}} = \frac{\mu_0}{4\pi}\int \frac{\vec{\mathbf{I}}(t_r)}{\mathscr{r}} dl = \frac{\mu_0 k}{4\pi}\int \frac{(t - \mathscr{r}/c)}{\mathscr{r}} d\vec{\mathbf{l}} = \frac{\mu_0 k}{4\pi}\left\{t\int\frac{d\vec{\mathbf{l}}}{\mathscr{r}} - \frac{1}{c}\int d\vec{\mathbf{l}}\right\}$ (1 point)

For the complete loop, $\oint d\vec{\mathbf{l}} = 0$ (1 point)

Note that this step can be done differently by direct integration of all four segments independently. If this is the case, a quarter of this point should be added to each of the four integrals below.

$$\vec{\mathbf{A}} = \frac{\mu_0 kt}{4\pi}\left\{\int_{Loop\ a}\frac{d\vec{\mathbf{l}}}{a} + \int_{Loop\ b}\frac{d\vec{\mathbf{l}}}{b} + 2\,\hat{\mathbf{x}}\int_a^b\frac{dx}{x}\right\}$$ (1 point)

Loop a (the lower semicircle):

$\int_{Loop\ a} d\vec{\mathbf{l}} = a\int_{-\pi}^{0}(-\sin\varphi\,\hat{\mathbf{x}} + \cos\varphi\,\hat{\mathbf{y}}) = a\,(\cos\varphi|_{\pi}^{0}\,\hat{\mathbf{x}} + \sin\varphi|_{\pi}^{0}\,\hat{\mathbf{y}}) = 2a\,\hat{\mathbf{x}}$ (2 points)

The outer semicircle: $\int_{Loop\ b} d\vec{\mathbf{l}} = b\cos\varphi|_0^{\pi}\,\hat{\mathbf{x}} = -2b\,\hat{\mathbf{x}}$ (1 point)

$$\vec{\mathbf{A}} = \frac{\mu_0 kt}{4\pi}\left\{\frac{2a}{a} - \frac{2b}{b} + 2ln\left(\frac{b}{a}\right)\right\}\hat{\mathbf{x}};$$ (1 point)

$$\vec{\mathbf{A}} = \frac{\mu_0 kt}{2\pi} ln(b/a)\,\hat{\mathbf{x}}$$

NB: the time in the first line should be **retarded** time. Curiously enough, this does not nesseseraly leads to the wrong answer as the retarded-time integral is zero (the second line above) but nonetheless is a mistake.

2. $V = 0;\ \vec{\mathbf{E}} = -\frac{\partial \vec{\mathbf{A}}}{\partial t} = -\frac{\mu_0 k}{2\pi} ln\left(\frac{b}{a}\right)\ \hat{\mathbf{x}}$ (1 point)

3. Similarily to $\vec{\mathbf{A}}$

$$\vec{\mathbf{B}}(\vec{\mathbf{r}}, t) = \frac{\mu_0}{4\pi}\int\left[\frac{\vec{\mathbf{I}}(\vec{\mathbf{r}}', t_r)}{\mathscr{r}^2} + \frac{\dot{\vec{\mathbf{I}}}(\vec{\mathbf{r}}', t_r)}{c\mathscr{r}}\right] dl \times \hat{\mathscr{r}}$$

$$= \frac{\mu_0 k}{4\pi}\int\left[\frac{\left(t - \frac{\mathscr{r}}{c}\right)}{\mathscr{r}^2} + \frac{1}{c\mathscr{r}}\right] d\vec{\mathbf{l}} \times \hat{\mathscr{r}}$$ (2 points)

$$= \frac{\mu_0 kt}{4\pi b}\int d\varphi\,\hat{\mathbf{z}}$$ (2 points: 1 point for integral, 1 point for the right direction)

$$= \frac{\mu_0 kt}{2b}\,\hat{\mathbf{z}} = \frac{\mu_0 I(t)}{2b}\,\hat{\mathbf{z}}$$ (1 point)

(5 points in total)

4. Because the time-derivative of current which *linearly* change in time, is perfectly compensated by retardation for which the Bio-Savert law doesn't account, either. (2 points)

<u>Typical mistakes:</u>

Q2.1. Not explicitly calculating at least one of the integrals for the loops: you must demonstrate how you arrived at this particular answer.

Q2.2 Occasionally forgetting the minus signs

Q2.2 There were also several vector signs on the V.

Q2.3 Carelessness with vector signs (many cross-products between scalars and vectors). Missing vectors, omitting the cross product, or coming up with incorrect quantities to replace $d\vec{\mathbf{l}}$.

Q2.3 Some copy-pasted Jefimenko's equation and stopped there. This does not count of course.

Q2.3 Some took the volume version of Jefimenko's equation (which is OK) but were not able to convert $\vec{\mathbf{J}}$ into $\vec{\mathbf{I}}$ (we did this at lectures).

Question 3. (10 points)

A 5G mobile network tower rises to height $h = 30\ m$ above flat horizontal ground (as is the case in the Netherlands). At the top is an ideal dipole emitter, which oscillated in the vertical direction (this is not exactly accurate but will do for the exam). The network broadcasts from this antenna with a total radiated power $P = 100\ W$ (that's averaged, of course, over a full cycle). Neighbours have complained about potential problems with the Covid-19 virus which they attribute to excessive radiation from the 5G tower. However, before setting up the 5G tower on fire, they hired you to assess if the maximum radiation intensity is in compliance with the radiation emission level of 1 mW/cm^2 (it is apparently *very* responsible neighbourhood).

1. In terms of the variables given, show that the intensity of the radiation at ground level, at a distance R from the base of the tower is

$$I(R) = \frac{3P}{8\pi}\frac{R^2}{(R^2 + h^2)^2}$$

(As usual, you may assume that $d \ll \lambda \ll r$. We are also interested only in the magnitude of the radiation, not in its direction – when measurements are taken, the detector will be aimed directly at the antenna.) (4 points)

2. How far from the base of the tower *should* you have made the measurement? First explain your reasoning without calculations. (1 point)

3. Now provide a formula of a distance from the base of the tower at which you made the measurement. (2 points)

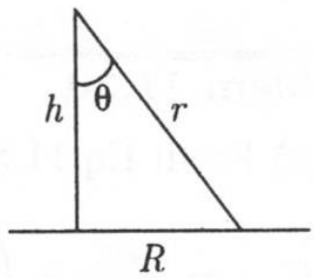
4. What is the formula for the intensity at this location? (1 point)

5. Is the radio emission of the tower in compliance? Provide the numbers to prove your answer. (2 points)

Answers to question 3 (Problem 11.23 modified, 10 points)

1. $\vec{\mathbf{I}} = \langle \vec{\mathbf{S}} \rangle = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c}\left(\frac{\sin^2\theta}{r^2}\right)\hat{\mathbf{r}}$ (1 point)

$\sin\theta = \frac{R}{r}; r = \sqrt{R^2 + h^2};\ \vec{\mathbf{I}} = \frac{\mu_0 p_0^2 \omega^4}{32\pi^2 c}\frac{R^2}{(R^2 + h^2)^2}\hat{\mathbf{r}}$ (2 points)



The total radiated power $P = \frac{\mu_0 p_0^2 \omega^4}{12\pi c}$ (1 point)

$$I(R) = \frac{12P}{32\pi}\frac{R^2}{(R^2 + h^2)^2} = \frac{3P}{8\pi}\frac{R^2}{(R^2 + h^2)^2}$$

2. The intensity just below the antenna is zero ($\theta = 0$ -- dipoles do not emit into the direction of their oscillation); the intensity is also low at $\theta \sim 90°$ because the distance is too large. Therefore, there should be a maximum. (1 point)

3. $$\frac{dI}{dR} = \frac{3P}{8\pi}\frac{2R(R^2 + h^2)^2 - R^2 2(R^2 + h^2)2R}{(R^2 + h^2)^4} = \frac{3P}{8\pi}\frac{R}{(R^2 + h^2)^3}(R^2 + h^2 - 2R^2) = 0$$

$R = h$ (2 points)

4. $I_{max} = \frac{3P}{8\pi}\frac{h^2}{(h^2+h^2)^2} = \frac{3P}{32\pi h^2}$ (1 point)

5. $I_{max} = \frac{3\ \cdot 100}{32\pi(30)^2} = 3.3 \cdot 10^{-3}\, W/m^2 = 3.3 \cdot 10^{-4}\,\frac{mW}{cm^2}$ (1 point)

Yes, the radiation emission level is *well* in compliance. (1 point)

Typical mistakes:

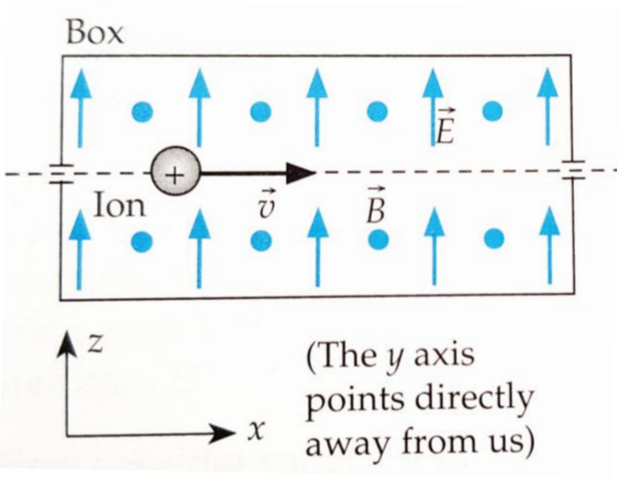
Q3.2-3 Coming out of the blue with 45 degrees without any explanation. Note that a reasoning as “it should not be 0 degree nor 90 so must be 45” is not correct in general.

Q3.4: Making calculation errors: forgetting the factor 2, forgetting to square the factor 2, dividing out the h^2’s entirely etc. However, if the error was minor (just a factor difference), we did not subtract points.

Q3.5: Doing incorrect unit conversions: e.g. dividing when going from 1/m^2 to 1/cm^2.

Question 4 (10 points)

The figure shows a schematic diagram of velocity selector, a device that allows a charged particle to pass through two coaxial (i.e. laying at the same axis) openings only if it has a specific velocity v which vector is also coaxial with the two openings. In the laboratory (box) system of coordinates, electric and magnetic fields are uniform inside the box and point into $+z$ and $-y$ directions, respectively. An ion with positive charge moves through the box in the +x direction with speed v. Gravity can be neglected.



1. Find electric and magnetic fields in the system of coordinates moving together with the charge. (4 points)

2. In the charge system of coordinates (i.e. if the observer sits on the ion), what are the components (electric or/and magnetic) of the Lorentz force that are exerted upon the charge? (1 points)

3. In the charge system of coordinates, what are requirements to the electric and magnetic fields to ensure the *straightforward* movement of the charge? (2 points)

4. Now we are back to the laboratory system. What are the components (electric or/and magnetic) of the Lorentz force that are exerted upon the charge? (1 point)

5. In the laboratory system, what are requirements to the electric and magnetic fields to ensure the *straightforward* movement of the charge? Explain why your answer makes sense. (2 points)

Answers to Question 4

1. $\vec{\mathbf{E}} = (0,0,E_z)$; $\vec{\mathbf{B}} = (0,-B_y,0)$

$\bar{E}_x = 0_x$; $\bar{E}_y = \gamma(0_y - v0_z)$; $\quad \bar{E}_z = \gamma(E_z - vB_y)$

$\bar{B}_x = 0_x$; $\bar{B}_y = \gamma\left(-B_y + \frac{v}{c^2}E_z\right)$; $\bar{B}_z = \gamma\left(B_z - \frac{v}{c^2}0_y\right)$

$\vec{\bar{\mathbf{E}}} = \left(0,0,\gamma(E_z - vB_y)\right)$; $\vec{\bar{\mathbf{B}}} = \left(0, -\gamma B_y + \gamma\frac{v}{c^2}E_z, 0\right)$

(4 points in total)

2. $\vec{\bar{\mathbf{F}}} = q\left(\vec{\bar{\mathbf{E}}} + \vec{\bar{\mathbf{v}}} \times \vec{\bar{\mathbf{B}}}\right)$

In the charge system, velocity is equal to zero $\vec{\bar{\mathbf{v}}} = 0$ so that only electric field component is exerted. (**1 point)**

3. For the straightforward movement, the total force must be equal zero so that

$\bar{\mathbf{E}} = \mathbf{0}$, $or\ E_z - vB_y = 0$, $or\ E_z = vB_y$ (**2 points)**

4. $\vec{\mathbf{F}} = q(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}})$ Both components are exerted (1 point)

5. The total force must be equal zero so that $E_z = vB_y$ (1 point)
Despite different forces exerted on the charge for two observers, they come to the same conclusion. (1 point)

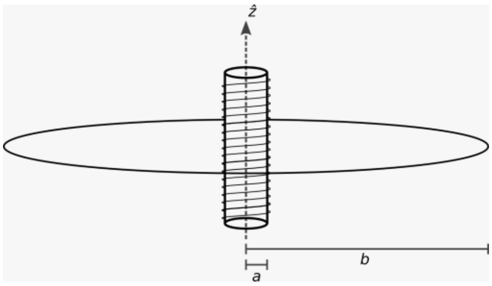
Typical mistakes:

4.1. Some students wrote $\vec{\mathbf{E}}$ and $\vec{\mathbf{B}}$ as oscillating fields as if it were some EM wave. Of xourse, there is no EM wave involved in this problem at all.

4.1. To find the electric and magnetic field in the frame of the ion, Example 10.4 was used. However, this describes the electric field of a uniformly moving charge while in the problem, we are asked to find the E and B fields acting on the moving ion. Therefore, we should use Eq. 12.109 to transform from the lab frame to the ion rest frame.

4.2. Sitting on the ion implies that $\vec{\bar{\mathbf{v}}} = 0$ in that frame, which was missed by some students.

Question 5. (15 points)



Consider a very long solenoid of radius a, with n turns per unit length, carries a current I_s running anticlockwise as seen from the positive z direction (see the figure). Coaxial with the solenoid, at radius $b \gg a$, is a circular ring of wire, with resistance R. The current in the solenoid is (gradually) decreased.

1. Show that the electric field right outside the solenoid is given as

$$\vec{\mathbf{E}} = -\frac{\mu_0 a n}{2}\frac{dI_s}{dt}\,\hat{\boldsymbol{\phi}} \quad \text{(5 points)}$$

2. Show that the magnetic field produced by the ring along its axis at a distance z from the center is given as

$$\vec{\mathbf{B}} = -\frac{\mu_0^2 \pi a^2 n}{2R}\frac{dI_s}{dt}\frac{b^2}{(b^2+z^2)^{3/2}}\hat{\mathbf{z}} \quad \text{(2 points)}$$

<u>Tip</u>: Use the following formula for the magnetic field along the axis of the ring at the distance z from the center (see Formula sheet or Eq.5.41):

$$\vec{\mathbf{B}} = \frac{\mu_0 I_r}{2}\frac{b^2}{(b^2+z^2)^{3/2}}\hat{\mathbf{z}}$$

3. Calculate the Poynting vector just outside the solenoid. Assume that the *electric* field $\vec{\mathbf{E}}$ is due to the changing flux in the solenoid while the *magnetic* field $\vec{\mathbf{B}}$ is due to the current in the ring I_r. As $b \gg a$, approximate the latter by its value along the axis. (2 points)

4. Calculate the power by integrating the Poynting vector over the entire surface of the solenoid. (4 points)

5. Now express your result is terms of the current in the ring I_r and its resistance R. Does your result make any sense? Explain why. (2 points)

Answers to question 5 (Problem 8.13 modified, 15 points)

1. $\oint \vec{\mathbf{E}} \cdot d\vec{\boldsymbol{l}} = -\frac{d\Phi}{dt}$ (1 point)

$$\frac{d\Phi}{dt} = \frac{d}{dt}(B\,\pi a^2) = \pi a^2 \frac{d}{dt}(\mu_0 n I_s) = \pi a^2 \mu_0 n \frac{dI_s}{dt} \quad \text{(2 points)}$$

The direction of $\vec{\mathbf{E}}$ is circumferential because $\vec{\mathbf{B}}$ is along the axis of the solenoid (1 point)

$$E\,2\pi a = -\pi a^2 \mu_0 n \frac{dI_s}{dt} \quad \text{(1 point)}$$

$$\vec{\mathbf{E}} = -\frac{\mu_0 a n}{2}\frac{dI_s}{dt}\,\hat{\boldsymbol{\phi}}$$

(5 points in total)

2. $\vec{\mathbf{B}} = \frac{\mu_0 I_r}{2}\frac{b^2}{(b^2+z^2)^{3/2}}\hat{\mathbf{z}}$

$$I_r = \frac{\mathcal{E}}{R} = -\frac{1}{R}\frac{d\Phi}{dt} = -\frac{\mu_0 \pi a^2 n}{R}\frac{dI_s}{dt} \quad \text{(2 points)}$$

$$\vec{\mathbf{B}} = -\frac{\mu_0^2 \pi a^2 n}{2R}\frac{dI_s}{dt}\frac{b^2}{(b^2+z^2)^{3/2}}\hat{\mathbf{z}}$$

3. $\vec{\mathbf{S}} = \frac{1}{\mu_0}(\vec{\mathbf{E}} \times \vec{\mathbf{B}}) = \frac{1}{\mu_0}\left(-\frac{\mu_0 an}{2}\frac{dI_s}{dt}\right)\left(-\frac{\mu_0^2 \pi a^2 n}{2R}\frac{dI_s}{dt}\frac{b^2}{(b^2+z^2)^{\frac{3}{2}}}\right)(\hat{\boldsymbol{\phi}} \times \hat{\mathbf{z}})$ (1 point)

$= \frac{\mu_0^2 \pi a^3 n^2}{4R}\left(\frac{dI_s}{dt}\right)^2 \frac{b^2}{(b^2+z^2)^{3/2}}\hat{\mathbf{s}}$ (1 point)

(2 points in total)

4. $P = \int \vec{\mathbf{S}} \cdot d\vec{\mathbf{a}} = \int_{-\infty}^{\infty} S\, 2\pi a\, dz$ (1 point)

$= \frac{\mu_0^2 \pi^2 a^4 n^2 b^2}{2R}\left(\frac{dI_s}{dt}\right)^2 \int_{-\infty}^{\infty} \frac{1}{(b^2+z^2)^{\frac{3}{2}}}\, dz$ (1 point)

$= \frac{\mu_0^2 \pi^2 a^4 n^2 b^2}{2R}\left(\frac{dI_s}{dt}\right)^2 \frac{2}{b^2} = \frac{\mu_0^2 \pi^2 a^4 n^2}{R}\left(\frac{dI_s}{dt}\right)^2$ (2 points)

(4 points in total)

5. $P = R\left(\frac{\mu_0 \pi a^2 n}{R}\frac{dI_s}{dt}\right)^2 = RI_r^2$ (1 point)

It is identical to the one predicted by Joule's heating low because of energy conservation law. (1 point)

<u>Typical mistakes:</u>

5. Some wrote I interchangeably with I_r, which makes the further solution a mess

5.3. $d^2 I_s/dt^2$ is not equal to $(dI_s/dt)^2$!

5.3. Missing the unit vector for the direction, or adding the one one where it shouldn't be there (e.g., $\vec{S} \cdot d\,\vec{a}$ is a scalar),

5.4 While integrating $P = \int \vec{\mathbf{S}} \cdot d\vec{\mathbf{a}}$, ignore the z-dependence in *S*, or that the integral goes over *dz*. Instead of doing the integration, the expression was multiplied by the area, introducing a height for the solenoid in the process.